سكشن اتنين

3 Duality

if $g(t) \rightleftharpoons G(f)$

then $G(t) \rightleftharpoons g(-f)$

Ex Find F.T of $A\tau \operatorname{sinc}(t\tau)$

using Duality

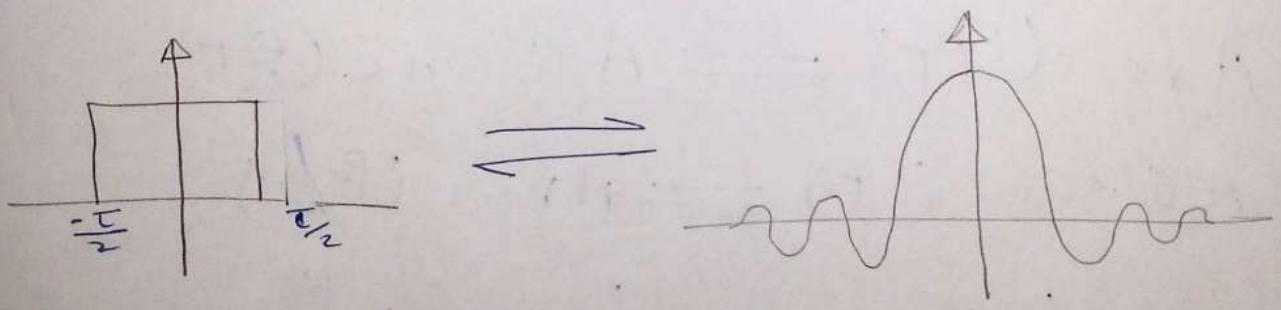
$A \operatorname{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau \operatorname{sinc}(f\tau)$

$A\tau \operatorname{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A \operatorname{rect}\left(\frac{-f}{\tau}\right)$

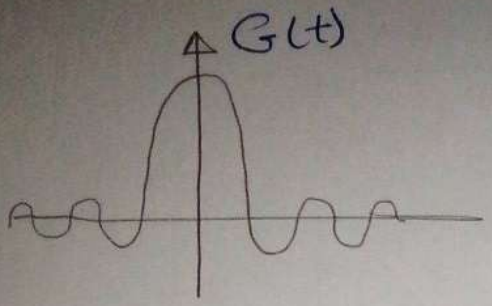
* في حالة ال rect فقط ممكن تشيل (-)

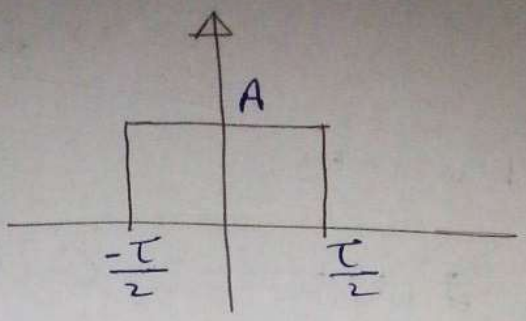
$-f=0, f=0$

كلاهما واحد



Limited ~~Freq~~ in time $\rightleftharpoons$ unlimited Freq





* using Duality

1) $A\,\text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau\,\text{sinc}(f\tau)$

2) $A\tau\,\text{sinc}(\tau t) \rightleftharpoons A\,\text{rect}\left(\frac{f}{\tau}\right)$

$A\sin(2\omega t) \rightleftharpoons \frac{A}{2\omega}\,\text{rect}\left(\frac{f}{2\omega}\right)$

$\tau = 2\omega$

EX Find F.T of $\sin c(mt)$ using Duality

$A\,\text{rect}(t/\tau) \rightleftharpoons A\tau\,\text{sinc}(f\tau)$

$A\tau\,\text{sinc}(t\tau) \rightleftharpoons A\,\text{rect}(f/\tau)$

$\text{sinc}(mt) \rightleftharpoons \frac{1}{m}\,\text{rect}\left(\frac{f}{m}\right)$

$\tau = m$, $A\tau = 1$, $A = \frac{1}{m}$

[4] Time shift Property

If $g(t) \rightleftharpoons G(f)$

then $g(t \pm t_0) \rightleftharpoons G(f) \cdot e^{\pm j2\pi f t_0}$

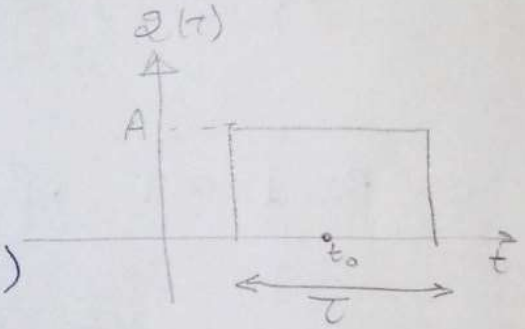
نفس زاوية الشاذة

Ex: Find F.T for $g(t) = A\,\text{rect}(t - t_0)$

amp

Sol

using time shift



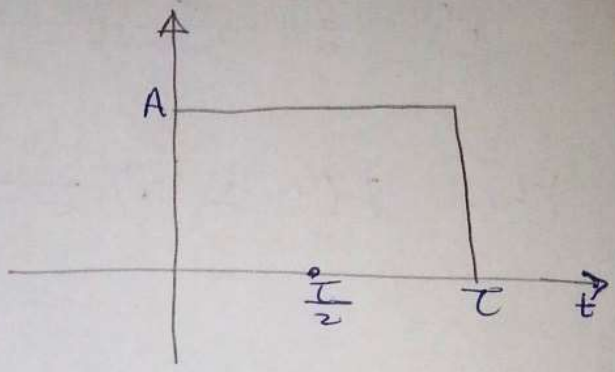
$$A\,\text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons \underbrace{A\tau \sin(f\tau)}_{G(f)}$$

$$A\,\text{rect}\left(\frac{t - t_0}{\tau}\right) \rightleftharpoons A\tau\,\text{sinc}(f\tau) \cdot e^{-j2\pi f t_0}$$

Ex Find F.T for $g(t) = A\,\text{rect}\left(\frac{t-\frac{\tau}{2}}{\tau}\right)$

Sol

using time shift



$$A\,\text{rect}\left(\frac{t-\frac{\tau}{2}}{\tau}\right) \rightleftharpoons A\tau\,\text{sinc}(f\tau)\cdot e^{-j2\pi f\frac{\tau}{2}}$$

$$A\tau\,\text{sinc}(f\tau)\cdot e^{-j\pi f\tau}$$

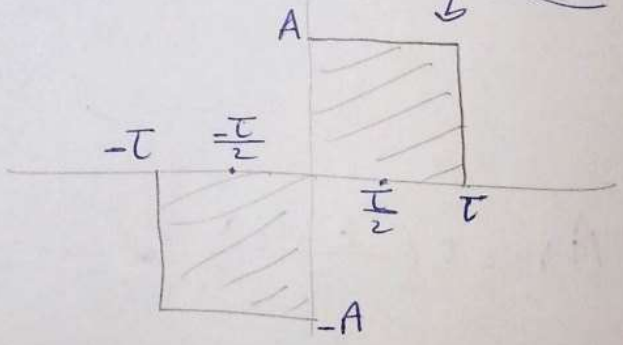
Ex Find F.T of $g(t)$ as shown.

$$g(t) = A\,\text{rect}\left(\frac{t-\tau/2}{\tau}\right)$$

$$-A\,\text{rect}\left(\frac{t+\tau/2}{\tau}\right)$$

~~$-A\,\text{rect}$~~

$A\,\text{rect}\left(\frac{t-\tau/2}{\tau}\right)$



using linearity & time shift

$$A\,\text{rect}\left(\frac{t-\tau/2}{\tau}\right) \rightleftharpoons A\tau\,\text{sinc}(f\tau)\cdot e^{-j2\pi f\frac{\tau}{2}}$$

$$A\,\text{rect}\left(\frac{t+\tau/2}{\tau}\right) \rightleftharpoons A\tau\,\text{sinc}(f\tau)\cdot e^{j2\pi f\frac{\tau}{2}}$$

$$G(f) = A\tau\,\text{sinc}(f\tau)\left[e^{-j\pi f\tau} - e^{+j\pi f\tau}\right]$$

$$\cdot * \frac{-2j}{-2j}$$

$$\therefore G(f) = (-2j)A\tau\,\text{sinc}(f\tau)\cdot\sin(\pi f\tau)$$

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5 Frequency-shift property

if $g(t) \rightleftharpoons G(f)$

then $g(t)\cdot e^{\pm j2\pi f t} \rightleftharpoons G(f \mp f_0)$

Ex Find F.T of $A\,\text{rect}\left(\frac{t}{\tau}\right)\cdot e^{-j2\pi f_c t}$

Sol

using freq. shift

$$A\,\text{rect}\left(\frac{t}{\tau}\right) \rightleftharpoons A\tau\,\text{sinc}(f\tau)$$

$$G(f) = A\tau\,\text{sinc}(f\tau)$$

$$G(f+f_0) = A\tau\,\text{sinc}(\tau(f+f_0))$$

Ex Find F.T of

$$g(t) = A\,\text{rect}\left(\frac{t}{\tau}\right)\cdot\cos(2\pi f_c t)$$

$$g(t) = A\,\text{rect}\left(\frac{t}{\tau}\right)\left[\frac{e^{j2\pi f_c t} + e^{-j2\pi f_c t}}{2}\right]$$

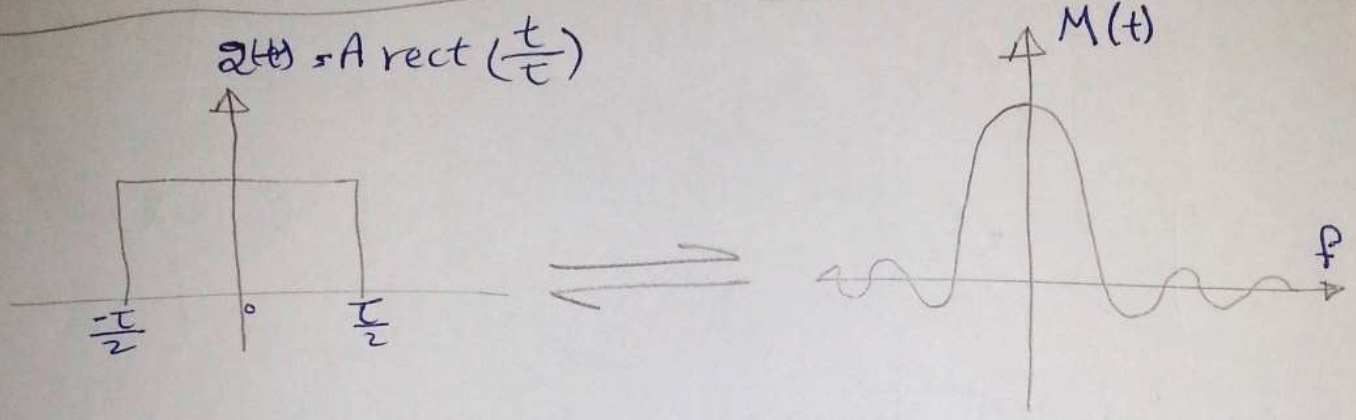
$$= \frac{1}{2}\left[A\,\text{rect}\left(\frac{t}{\tau}\right)\cdot e^{j2\pi f_c t} + A\,\text{rect}\left(\frac{t}{\tau}\right) e^{-j2\pi f_c t}\right]$$

using superposition & Linearity

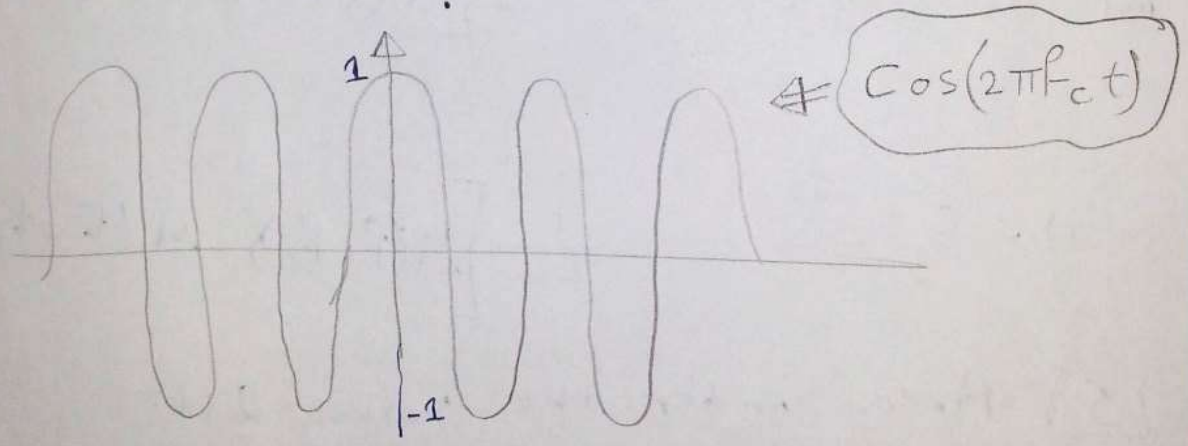
$$A\,\text{rect}\left(\frac{t}{\tau}\right)\cdot e^{j2\pi f_c t} \rightleftharpoons A\tau\,\text{sinc}((f-f_c)\tau)$$

$$A\,\text{rect}\left(\frac{t}{\tau}\right)\cdot e^{-j2\pi f_c t} \rightleftharpoons A\tau\cdot \text{sinc}\left((f+f_c)\tau\right)$$

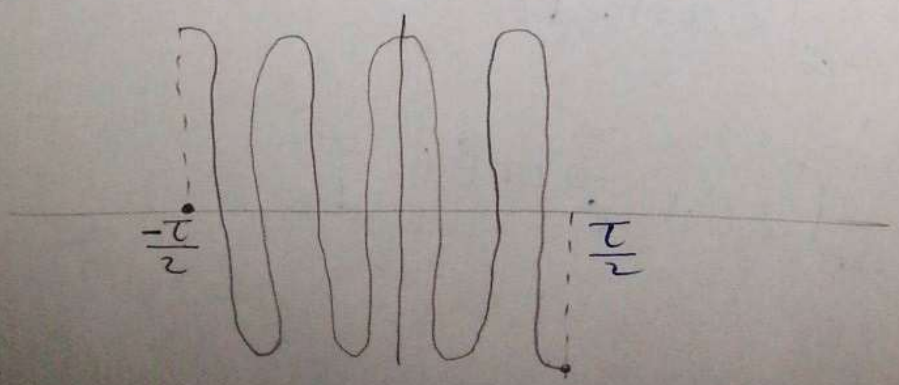
$$\therefore G(f) = \frac{1}{2}\left[A\tau\cdot\text{sinc}\left((f-f_c)\tau\right) + A\tau\,\text{sinc}\left((f+f_c)\tau\right)\right]$$



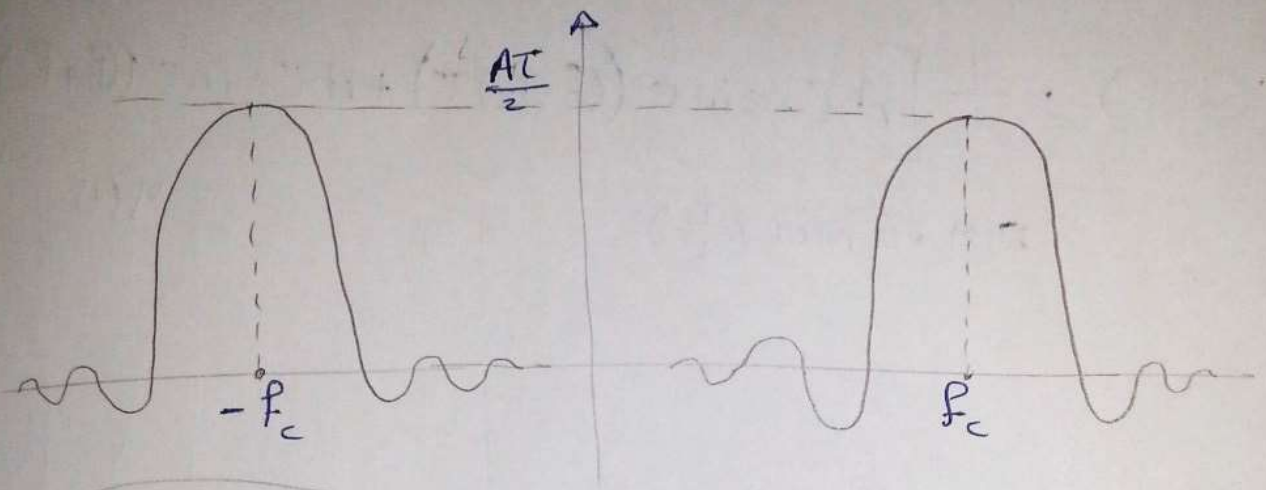
$$q(t) = A\,\text{rect}\left(\frac{t}{\tau}\right)\cdot\cos(2\pi f_c t)$$



$q(t)$



$$G(f) = \frac{1}{2}\left[A\tau \,\text{sinc}((f-f_c)\tau) + A\tau \,\text{sinc}((f+f_c)\tau)\right]$$



* Modulation Theory :-

$$m(t)\cdot\cos(2\pi f_c t) \rightleftharpoons \frac{1}{2}\left[M(f+f_c) + M(f-f_c)\right]$$

$$m(t)\cdot\frac{e^{j\theta} - e^{-j\theta}}{2\tau} \rightleftharpoons \frac{1}{2\tau}\left[M(f-f_c) - M(f+f_c)\right]$$

[6] Area under the curve $g(t)$

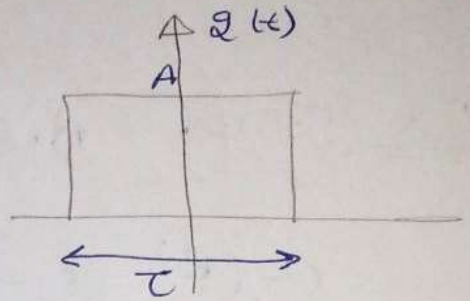
$$\text{Area} = \int_{-\infty}^{\infty} g(t)\cdot dt$$

$$G(f) = \int_{-\infty}^{\infty} g(t)\cdot e^{-j2\pi f t}\cdot dt$$

$$\therefore \text{Area} = G(0)\Big|_{f=0}$$

Ex Find Area of $g(t) = A\,\text{rect}\left(\frac{t}{\tau}\right)$

Sol



$$\text{Area} = A * \tau$$

$$\text{Area} = A\tau$$

another solution

$$G(f) = A\tau\,\text{sinc}(f\tau)$$

$$\text{Area} = G(0) = A\tau\,\underbrace{\text{sinc}(0)}_{\to 1} = A\tau$$

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[7] Area under Curve $G(f)$

$$\text{Area} = \int_{-\infty}^{\infty} G(f)\,df$$

I.F.T

$$g(t) = \int_{-\infty}^{\infty} G(f)\cdot e^{+j2\pi f\cdot t}\cdot df$$

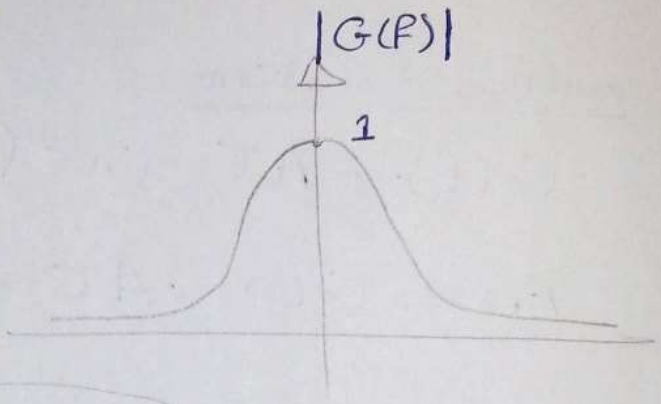
$\Rightarrow t = 0$

$\therefore$ Area $= g(0)$

Ex Find Area under Curve

$$G(f) = \frac{1}{j2\pi f}$$

$$|G(f)| = \frac{1}{\sqrt{1+4\pi^2 f^2}}$$



~~g(t)~~

$g(t) = e^{-t} \cdot u(t)$

$\therefore$ Area $= g(0)$

Area $= 1 \cdot \frac{1}{2}$

$= \frac{1}{2}$

Report 2

5 Find F.T for

1. $g(t) = 3\,sgn(t-3)$
2. $g(t) = rect\left(\frac{t+1.5}{7}\right)$
3. $g(t) = 3 \cdot e^{-|t-2|}$

10 sec 4